Department of Mathematics

Syllabus for

Master of Science in Applied Mathematics (MSAM)

under Weekend Program

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under Weekend Program

2024 Onwards

Department of Mathematics
Jahangirnagar University

Overview of the Master of Science in Applied Mathematics (MSAM):

- **Class day:** Friday and/or Saturday
- **Duration of each semester:** 4 (Four) Months
- **Course work:** 9×3 = 27 Credit hours
- **Viva-Voce + Project:** (1.5+1.5=) 3 Credit hours
- **Total credit hours:** 10×3 = 30 Credit hours
- **Marks distribution of each course:**

Continuous Assessment	Tutorial	10	40
	Mid-Term	20	
	Attendance	10	
Final Examination			60
Total			**100**

- **Grading Policy:**

Conversion Point	Letter Grade	Grade Point
80-100	A+	4.00
75-less than 80	A	3.75
70-less than 75	A-	3.50
65- less than 70	B+	3.25
60- less than 65	B	3.00
55- less than 60	B-	2.75
50- less than 55	C+	2.50
45- less than 50	C	2.25
40- less than 45	D	2.00
below 40	F	00

- **Degree requirements:**

a) Students having BSc (Hons) in Mathematics or Applied Mathematics, the degree will be awarded after the successful completion of **30** credit hours.
b) Students having BSc (Eng.) or BSc (Hons) in Physical and Mathematical Sciences must complete ***3 preliminary courses***, each of 3 credits in addition to the above **30** credits courses in (a).
c) Student having BSc (pass) with Mathematics or BSc (Hons) in Biological Sciences must complete ***6 preliminary courses***, each of 3 credits in addition to the above **30** credits courses in (a).
d) Student having BSc (pass) with Mathematics and having done preliminary courses of Masters level in Mathematics shall come under category (a).
e) Credits in preliminary courses in **MSAM program** shall be shown in the transcripts but not be included in the calculation of CGPA.
f) Students must pass all the courses individually by getting at least “**D**” grade and maintaining a minimum **CGPA 2.50**.

Core Courses

Course Code	Course Title	Marks	Unit	Credit hours
AMath 501	Analytical Methods of Partial Differential Equations	100	1	3
AMath 502	Qualitative Analysis of Ordinary Differential Equations	100	1	3
AMath 503	Applied Business Mathematics	100	1	3
AMath 504	Operations Management	100	1	3
AMath 505	Operations Research	100	1	3
AMath 506	Advanced Fluid Dynamics	100	1	3
AMath 507	Computational Fluid Dynamics	100	1	3
AMath 508	Nonlinear Dynamics and Chaos	100	1	3
AMath 509	Mathematical Biology	100	1	3
AMath 510	Advanced Numerical Methods	100	1	3
AMath 511	Scientific Computing	100	1	3
AMath 512	Graph Theory with Algorithms	100	1	3
AMath 513	Fuzzy Mathematics with Applications	100	1	3
AMath 514	Lattice Theory and Applications	100	1	3
AMath 515	Mathematical Statistics	100	1	3
AMath 516	Machine Learning	100	1	3
AMath 517	Data Analytics	100	1	3
AMath 518	Integral Equations with Applications	100	1	3

Preliminary Courses

Course Code	Course Title	Marks	Unit	Credit hours
PAMath 519	Theory of Number Systems	100	1	3
PAMath 520	Calculus with Analytical Geometry	100	1	3
PAMath 521	Linear Algebra	100	1	3
PAMath 522	Ordinary Differential Equations	100	1	3
PAMath 523	Partial Differential Equations	100	1	3
PAMath 524	Special Functions and Integral Transforms	100	1	3

Project Report and Viva voce

Course Code	Course Title	Marks	Unit	Credit hours
AMath 530	Report	50	1/2	1.5
AMath 531	Viva voce	50	1/2	1.5

DETAILS OF THE COURSES

Core Courses

AMath 501: Analytical Methods of Partial Differential Equations

Learning Objectives:

- To acquire the ability to solve various types of PDEs using techniques such as separation of variables, the method of characteristics, Fourier and Laplace transforms, and numerical methods.
- To interpret PDE solutions within the context of physical or mathematical scenarios, recognizing the significance of boundary and initial conditions.
- To gain the ability to formulate PDEs for representing real-life problems in engineering, science, and other fields, taking into consideration the appropriate boundary and initial conditions.
- To apply these PDE solutions for analyzing and predicting behavior across diverse areas, including traffic flow, heat diffusion, wave propagation, population dynamics, quantum mechanics, and more.

Course Content:

1. ***PDE as models of physical problems:*** Traffic flow model, Conservation laws, heat equation, Poisson equation, shallow water waves, PDEs in the Life Science-Age structured population model, Advection-Diffusion equation, Burgers' equation, PDE in Vibration and Acoustics – Wave equation, PDE in Quantum mechanics – Schroedinger equation classification of PDE's. Well-posed problem, the Cauchy-Kovalevaskaya theorem.
2. ***Hyperbolic PDEs:*** first order equations, Cauchy problem, method of characteristics, the wave equations, shock waves in scalar conservation laws, weak solutions.
3. ***Elliptic PDEs:*** classical solutions, maximum principles, continuous dependence on the data (stability), existence and uniqueness of Dirichlet problem. Varational formulation, weak solution, the Lax-Miligram theorem.
4. ***Parabolic Equations:*** Physical examples of diffusion, heat flow, convective mixing in porous media, theory of linear parabolic equations- Fourier transform methods, unbounded domains, maximum principle, bounded domains, uniqueness and stability.

Suggested Readings:

1. J. David Logan (2014), *Applied Partial differential equations*, Springer.
2. Michael Renardy and Robert C. Rogers (2004), *An Introduction to Partial Differential equations*, Springer.
3. F. John (1981), *Partial differential equations*, Springer.
4. R. J. Leveque (1990), *Numerical methods for conservation laws,* Birkhauser
5. W. Hackbusch (2017), *Elliptic differential equation- theory and numerical Treatment*, Springer-Verlag.
6. K. Sankara Rao (2010), *Introduction to Partial Differential Equation*, PHI.

AMath 502: Qualitative Analysis of Ordinary Differential Equations

Learning Objectives:

- To understand the existence and uniqueness of the solutions of differential equations without solving the differential equation.
- To understand the nature of the solutions near the equilibrium solution of the system of differential equations.
- To understand the stability of the solutions of the system of differential equations.
- To understand various application in the field of engineering, mathematical physics, mathematical biology and physiology, fluid dynamics, economics etc.

Course Content:

1. ***Mathematical Modelling:*** ODEs as Mathematical Models of Physical Problems.
2. ***Existence-Uniqueness**:* Picard-Lindeloef existence uniqueness theorem, Peano existence theorem, Grounwall lemma.
3. ***Phase line analysis**:* critical point, equilibrium solution and their classification.
4. ***Phase plane analysis:*** Autonomous system, phase flow, orbit, phase-portrait,
5. ***Linearization:*** Method of linearization of non-linear system, periodic solution.
6. ***Linear System:*** Eigen-value method for solving linear system, Phase portrait.

7. ***Exponential ansatz in Linear Systems:*** Exponential ansatz and matrix method for homogeneous linear system. Calculation of Fundamental matrix, solution of linear system as an initial value problem.
8. ***Stability:*** Stability of solutions, Lyapunov stability, stability of periodic solution.

Suggested Readings:

1. M. Braun (1992), *Differential Equations and their Applications*, Springer.
2. Wolfgang Walter (1998), *Ordinary differential equations (translated by R Thomson)*, Springer.
3. Dennis G. Zill (2012), *A First Course in Differential Equations with Modeling and Applications*, Cengage Learning
4. Shepley L. Ross (2007), *Differential Equations*, John Wiley & Sons.

AMath 503: Applied Business Mathematics

Learning Objectives:

- To solve the business problems in the area of social science finance, accounting, economics, production and general management, and sales and marketing.
- To use differential and integral calculus for a wide range of applications like optimization, calculation of regular and irregular areas, curve sketching, maximization and minimization problems etc.
- To learn various tools and techniques so that the students will be able to solve the various real life problems related to social science, business, management and economics.

Course Content:

1. ***Theory of sets and its applications:*** Basic concept in real number system, set and functions, Venn diagram, applications of set theory in business and economics.
2. ***Linear Equations and Functions:*** Concept of linear equations, coordinates, independent and dependent variables, slopes, intercepts. Parallel and perpendicular lines, revenue, cost and profit functions, Break-even analysis, Linear Demand Functions, Applications problems, Piece-wise linear functions and its real-life applications.
3. ***System of Linear Equations:*** Several applications of the system of linear equations such as Mixture problems, supply and demand analysis, two-product supply and demand analysis etc.
4. ***Matrix Algebra and its Applications:*** Leontief Input-Output Models, Markov Chain, National Income Model, Cost, Revenue, Profit functions.
5. ***Time Value of Money:*** Basic concepts on exponential and logarithmic functions. Simple interest and future value, Effective rate: simple interest, compound interest rate and the future value, the conversion period, Compound discount: present value, effective rate: compound interest, Ordinary annuities, Present values and future values, Sinking fund payment, Amortization payment, Deferred annuity
6. ***Applications of Differential Calculus:*** Optimization of differential problems (concept of stationary points, local maximum and local minimum, end point maximum and minimum), Finding coordinates of all optimum points using first, second and third derivatives, Application problems. Marginal Propensity to consume and the Multipliers. Calculus of two independent variables, Maxima and Minima of a function of two independent variables and their applications.
7. ***Applications of Integral Calculus:*** Determination of total cost functions using rate function. Applications related to exponential rate functions. Area between two curves, Demand curve and supply curve. Finding Consumers' surplus and Producers' surplus using demand and supply law. Investment analysis, Continuous money flow, Total income, etc.

Suggested Readings:

1. Gordon D. Prichett, John C. Saber, and Earl K. Bowen (2006), *Mathematics with Applications in Management and Economics*. Irwin, Sydney, Australia.
2. Md. Rafiqul Islam and Mohammad Osman Gani (2022). *Business Mathematics*. CBO Printing and Publications, Bangladesh.

AMath 504: Operations Management

Learning Objectives:

- To understand the input-process-output framework, and apply them to a wide range of operations
- To examine the type of transformation processes occurring within operations.
- To define the role and responsibilities of operations managers and the challenges they face.
- To understand the contents of an operations strategy and the decisions involved.

Course Content:

1. ***Introduction:*** Introduction to Operations Management (OM), The scope of OM, OM and decision making productivity, Product mix, Strategy, Competitiveness.
2. ***Forecasting:*** Features common to all forecasts, Elements of good forecast, steps in the forecasting process, Accuracy and control of forecasting, Applications forecasting models.
3. ***Capacity Planning:*** Strategic capacity decision, Strategy formulation, Defining and measuring capacity, Evaluating capacity alternatives.
4. ***Quality Control:*** Management of quality, Statistical process control, Variations and control, Control charts, Process capability, Improving process capability, Capability analysis.
5. ***Inventory Control:*** Nature and importance of inventories, Introduction to basic inventory models (Economic order quantity (EOQ) model, EPQ model, fixed order interval model, Single period model.)
6. ***Scheduling:*** Scheduling in high-volume systems, intermediate-volume systems, low-volume systems, Scheduling methods of Linear Programming, Scheduling jobs through two work centers, minimizing scheduling difficulties, scheduling the work force.
7. ***Project Management:*** Behavioral aspects of project management, Key decisions in project management, PERT (Program Evaluation and Review Technique), CPM (critical pert method), Deterministic time estimates, Probabilistic time estimates, Applications.
8. ***Supply Chain Management:*** Foundation and Framework, Overview of supply chain management, strategic dimensions of supply chains, global dimensions, relationships, performance measurement and financial analysis.

Suggested Readings:

1. William J. Stevenson (2017), *Operations Management*, McGrew-Hill Higher Education
2. Wayne L. Winston (2004), *Operations Research*, Thomson Books/Cole
3. Hiller and Lieberman (2010), *Introduction to Operations Research*, McGrew-Hill Higher Education

AMath 505: Operations Research

Learning Objectives:

- To learn solving procedures of problems in different environments that needs decisions.
- To introduce application of deterministic and stochastic approaches and techniques for effective decision making.
- To formulate model and applications occurring in business, economic and management problems.

Course Content:

1. ***Basics of Operations Research(OR):*** Definitions, characteristics, necessity, classification and scope of OR, modeling and methods of OR, applications and limitations of OR.
2. ***Transportation and Assignment Problems****:* Mathematical formulation, relationship with linear programming problem, solution procedure and applications.
3. ***Game Theory****:* Two person zero sum game, maximin-minimax pure strategies, mixed strategies and expected payoff, games without saddle point, graphical method, dominance principle, symmetric game, solution of m×n game by linear programming and Brown’s algorithm.
4. ***Decision Analysis****:* Decision alternatives, states of nature or events, pay-off, decision making under certainty, decision making under uncertainty, decision making under risk, decision making under conflict, use of subjective probabilities in decision making, use of posterior probabilities in decision making (Bayesian Analysis), decision tree analysis.
5. ***Markov Chains****:* Characteristics of Markov chain, state and transition probabilities, transition probability matrix, steady state conditions, absorbing state, transient state, recurrent state, application of Markov chain in business problems.

6. ***Poisson Process:*** Inter-arrival and waiting time distributions; conditional distribution of the arrival times, non-homogeneous Poisson process; compound Poisson process, conditional Poisson process.
7. ***Queuing theory:*** Characteristics of queuing system, probability distributions in queuing system, single server queuing models, multi-server queuing models, birth and death process.
8. ***Simulation:*** Basic terminology of simulation, steps in simulation process, Application of simulation, Simulation with random variables, Advantage and limitations of using simulations.

Suggested Readings:

1. J. K. Sharma (2006), *Operations Research : Theory and Applications*, MACIN
2. A. M. Natarajan, P. Balasubramani, A. Tamilarasi (2009), *Operations Research*, Dorling Kindersley
3. C. R. Kothari (2018), *An introduction to Operational Research*, Vikash Publishing House
4. S. R. Yadav, A.K. Malik (2014), *Operations Research*, Oxford University press.

AMath 506: Advanced Fluid Dynamics

Learning Objectives:

- To develop model of fluid dynamical problems, say, any sort of biological flow, blood flow, flow along lung and flow through mechanical device.
- To describe the non-linear PDE's such as mathematical biology, chemical flow through organs, etc.
- To learn the behavior of complex flow, say, turbulent flow for unsteady and high Reynold's number, Tsunami and ocean waves and so on.

Course Content:

1. **Introduction:** Types of different flows, different forces, normal stress and shearing stress, Newton's law of viscosity, Hagen Poiseuille equation of flow, Material and convective derivatives.
2. **Governing Equations of Fluid Flow**: Conservation of fluid flow, momentum equations of flow dynamics, energy equations of flow domain, stress and strain analysis for deformable fluid body, Stokes hypothesis.
3. **Exact solutions of NSEs:** Limitations of NSEs, Parallel flow, Poiseuille flow, Couette flow, flow for accelerated channel, flow for oscillating plate, creeping flow.
4. **Boundary layer:** Concept of boundary layer, boundary layer equations, Blasius equations, Solution of boundary layer equations, Prandtl's boundary layer equation, Reynolds principle of similarity.
5. **Dimensional Analysis:** Concept of dimensional analysis, the Reynolds effect on fluid flow, Buckingham's pi-theorem, application of pi-theorem, Rayleigh's method, similitude and modeling, physical importance of non-dimensional parameters, lift and drag coefficients, skin friction coefficients, flow over a flat plate, free convection boundary layer flow.
6. **Turbulent Flows:** Reynolds average principles, continuity equation of turbulent flow, momentum equation of turbulent low, Reynold's stress, eddy viscosity, Prandtl's mixing length theory, Prandtl's velocity distribution theory.

Suggested Readings:

1. Hermann Schlichting (2017), *Boundary Layer Theory*, Springer
2. Frank M. White (2011), *Fluid Mechanics*, Mc Graw-Hill
3. Frank Chorlton (1967), *Textbook of Fluid Dynamics*, Van Nostrand Company

AMath 507: Computational Fluid Dynamics

Learning Objectives:

- To predict and understand how fluids, such as air or water move and interact in various physical systems.
- To analyze the transfer of heat in different scenarios, such as in heat exchangers, electronic cooling systems, and industrial furnaces.
- To provide visual insights into complex flow patterns that are often challenging to observe experimentally.
- To optimize the design of engineering components and systems.

Course Content:

1. **Governing Equations for CFD:** Introduction, finite control volume, infinitesimal fluid element, Stokes' First problem, Stokes' Second problem, substantial derivative, the physical interpretations of the continuity equation, the momentum equation and the energy equation, the physical boundary conditions, problems for incompressible flow, the pipe flow.
2. **Finite Difference Method (FDM):** Simple method, general method and multidimensional finite difference formula, polynomial method for uniform and non-uniform grids.
3. **Parabolic Equations:** Explicit methods, FTCS method, DuFort-Frankel method, Implicit methods, Crank-Nicolson method, Beta formulation, applications.
4. **Analysis of Finite Difference Equations:** Consistency, stability analysis and convergence, Von Neumann stability analysis, error analysis.
5. **Elliptic Equations:** Jacobi iteration method, Point successive over-relaxation method (PSOR), Line successive over-relaxation method (LSOR), Alternating direction implicit method (ADI).
6. **Finite Element Method (FEM):** Basics of FEM, shape functions, one dimensional and two dimensional linear elements, one dimensional quadratic element, two dimensional quadrilateral elements, Rayleigh- Ritz method (variational method), method of weighted residuals, Galerkin finite element method.

Suggested Readings:

1. J. D. Anderson (1995), *Computational Fluid Dynamics*, McGraw-Hill
2. R. W. Lewis (2015), *Fundamentals of the Finite Element Method for Heat and Fluid Flow*, WILEY
3. T. J. Chung (2002), *Computational Fluid Dynamics*, Cambridge University Press
4. K. A. Hoffmann (2000), *Computational Fluid Dynamics Volume-I*, Engineering Education System, USA

AMath 508: Nonlinear Dynamics and Chaos

Learning Objectives:

- To derive differential equations that properly represent classical dynamic systems.
- To identify the expected equilibrium points and to assess the corresponding stability.
- To assess solutions of differential equations qualitatively, which are hardly accessible otherwise and understand the related physical meaning and implications.

Course Content:

1. ***One-Dimensional Flow:*** Flow on the lines, fixed point and stability, linear stability analysis, existence and uniqueness, Flows on the circles.
2. ***Bifurcations in One Dimension:*** An introduction to bifurcations, different types of bifurcations.

3. Linear Two-Dimensional Flow: Linear systems, classification of linear systems. Phase plane, phase portraits, fixed points and their stability.

4. Nonlinear Two-dimensional Flow: Linearization, reversible systems, pendulum, Limit cycles, van der Pol oscillator, Liénard systems, relaxation oscillations. Bifurcation theory in two-dimensional systems.

5. *Chaos:* Lorentz equations. One-Dimensional maps. Fractals.

Suggested Readings:

1. S. H. Strogatz (2001). *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*. Westview press.
2. Yuri A. Kuznetsov (2004). *Elements of Applied Bifurcation Theory*, Springer.

AMath 509: Mathematical Biology

Learning Objectives:

- To formulate and solve mathematical models of evolution in terms of optimization.
- To use techniques from ordinary differential equations to describe population genetics.
- To use techniques from partial differential equations to describe spread of genes, disease and other biological material.
- To explain how these techniques are applied in scientific studies and applied in ecology and epidemiology.

Course Content:

1. ***Population dynamics:*** Single species and multispecies ecosystems, competition, mutualism, growth and spatial spread, Fisher's equation.
2. ***Epidemiology:*** The spread of epidemic diseases, SIR model, SIER and SISER model.
3. ***Reaction-Diffusion models:*** Turing mechanism for pattern formation. How the leopard got his spots (and sometimes stripes), biological waves, Chemotaxis.
4. ***Enzyme Kinetics and chemical reactions:*** Enzyme reaction kinetics, molecular motors, Michaelis-Menten theory, hormone cycles, neuron-firing.
5. ***Mass transport:*** Mass transport process, membrane transport process, Taylor dispersion.
6. ***Biomechanics:*** Blood circulation, animal locomotion: swimming flight. Effects of scale and size.

Suggested Readings:

1. J. D. Murray (2002), *Mathematical Biology I. An Introduction*, Springer.
2. Leah Edelstein-Keshet (2005), *Mathematical Models in Biology*, SIAM.
3. Nicholas F. Britton (2003), *Essential Mathematical Biology*, Springer London.

AMath 510: Advanced Numerical Methods

Learning Objectives:

- To learn various tools and techniques to solve given problems in Mathematical Sciences.
- To impart the basic theories and fundamentals of numerical methods.
- To acquire skills to implement the mathematical formulation of various methods for computer solution.

Course Content:

1. Polynomial interpolation- the Vandermonde approach, the Newton representation; piecewise polynomial interpolation, cubic splines.
2. Numerical Integration: Adaptive quadrature.
3. Matrix computation: Gaussian Elimination and its variants, sensitivity of linear System, condition number, stability analysis, the least square method, Eigen vector, Eigen-value problems and Cholesky factorizations.
4. Iterative methods for linear system: The classical methods of linear system, convergence of iterative method, steepest descent method, the conjugate gradient method.
5. Nonlinear equations and optimization: Finding roots, minimization of a function solving system of non-linear equations.
6. Curve Fitting, Numerical Optimization.

Suggested Readings:

1. Charles F. Van Loan (1999)**,** *Introduction to scientific computing*, Prentice-Hall Inc., Pearson.
2. David S. Watkins (2010), *Fundamentals of Matrix Computations*, Wiley.
3. Burden and Faires (2015), *Numerical Analysis*, Cengage Learning.
4. J. H. Mathews and K. D. Fink (2003), *Numerical Methods using MATLAB*, Pearson.

AMath 511: Scientific Computing

Learning Objectives:

- To introduce numeric and algorithmic techniques used for the solution of a broad range of real-world mathematical problems in science and engineering.
- To learn numerical implementation of different techniques for solving various applied problems.
- To develop skills on one or more array-oriented numeric programming environments: MATLAB, Python, or some similar package.

Course Content:

1. ***Introduction to Computer Arithmetic:*** Floating point arithmetic, conditioning, stability, discretization, discretization error, accuracy and order of accuracy, local and global error, efficiency and convergence, spectral theory of matrices.
2. ***Eigenvalues and Singular Values:*** Matrix eigenvalue problems, singular value decomposition (SVD), power method, Jacobi method, unconstrained optimization problems, golden ratio search method.
3. ***Spectral Methods*****:** The Method of Weighted Residuals, Galerkin and Collocation approximation of a function, Discrete Fourier transform (DFT), Fast Fourier transform (FFT), smoothness and spectral accuracy of DFT and FFT.
4. ***Solution to Ordinary Differential Equations (ODEs):*** Runge-Kutta fourth order method, Adams-Bashforth method, Adams-Moultron method, shooting method, finite difference method, collocation method, one-step and multi-step methods for stiff systems of ordinary differential equations and differential-algebraic equations.
5. ***Solution to Stochastic ODEs:*** Stochastic modeling, stochastic simulation.

Suggested Readings:

1. Cleve B. Moler (2008), *Numerical Computing with MATLAB*, SIAM
2. Richard L. Burden and Douglas J. Faires (2016), *Numerical Analysis*, Cengage Learning
3. Steven Chapra and David Clough (2021), *Applied Numerical Methods with Python*, McGraw Hill

AMath 512: Graph Theory with Algorithms

Learning Objectives:

- To understand and apply the fundamental concepts in graph theory.
- To apply graph theory based tools in solving practical problems, e.g. shortest path, minimum spanning trees, etc.
- To improve the proof writing skills.

Course Content:

1. ***Preliminaries:*** Graphs, Different types of graphs, Graph isomorphism, Subgraphs, Operations on graphs.
Connected and Weighted Graphs: Connected Graphs, Weighted graphs, weighted connected graphs, Walks, Trails, Paths, Circuits, Graph distance, Cut-vertices and Cut-edges, Connectivity. Bipartite graphs, m-partite graphs; Dense and Sparse graphs.
Undirected Graphs: Degree of a vertex, Pendant and Isolated vertices, Maximum and Minimum degree of graphs. Degree sequences of graphs, The Handshaking Theorem, Adjacent List and Adjacent Matrix representation. **Directed Graphs (Digraphs):** In-degree and out-degree of a vertex, Adjacent List and Adjacent Matrix representation.

2. ***Trees and Spanning Trees:*** Basic definitions, Characterizations, Properties and use of trees, Acyclic graphs, Rooted and Binary trees. Depth-First Search (DFS) algorithm and Bredth-First Search (BFS) algorithm. Spanning trees, Chyley's theorem. Spanning tree algorithm, Kirchoff's algorithm.

3. ***Minimum Spanning Trees:*** Basic definitions. Minimum spanning tree algorithms with applications: Kruskal's algorithm, Prim's algorithm, Boruvka-Sollin's algorithm and Reverse deletion algorithm.

4. ***Shortest paths:*** Basic definitions. Shortest paths algorithms with applications:
Single pair shortest paths: The Bellman-Ford's algorithm, Dijkstra's algorithm, Johnson's algorithm.
All pair shortest paths: Floyd-Warshall's algorithm, Johnson's algorithm.
Linear programming (LP) formulation of the shortest path problem.

5. ***Eulerian and Hamiltonian Graphs:*** Euler Path, Euler circuit (Tour) and Eulerian graphs. Fleury's algorithm, Euler's path and Euler's Circuit theorem. Hamilton path, Hamilton circuit and Hamiltonian graphs.
Chinese Postman Problem (CPP). Travelling Salesman Problem (TSP): Nearest Neighbor algorithm, sorted edges (Cheapest link) algorithm and Lower Bound algorithm.

6. ***Matching, Graph colorings and Planar Graphs****:* Basic Definitions. Vertex coloring, Edge coloring, Coloring algorithms. Euler's formula for planar graphs. Maximal planar graphs. The four color theorem. Applications in scheduling.

7. ***Network Flows and Applications:*** Basic definitions. Max-flow Min-cut theorem. Network flows with anti-parallel edges, Network flows with multi-sources and multi-sinks. Maximum flow algorithms with applications: Ford-Fulkerson algorithm, Edmonds-Karp algorithm. Linear programming formulation of Maximal Flow Problem, Bounded variable simplex method.

Suggested Readings:

1. Alan Gibbons (1985), *Algorithmic Graph Theory*, Cambridge University Press
2. Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein (2009), *Introduction to Algorithms*, MIT press
3. Douglas B. West (2000), *Introduction to Graph Theory*, Pearson College Div
4. G. Suresh Singh (2010), *Graph Theory*, Prentice Hall of India

AMath 513: Fuzzy Mathematics with Applications

Learning Objectives:

- To know the fundamentals of fuzzy set theory.
- To know the basic definitions, theory and properties of fuzzy algebra
- To know the basic idea of fuzzy topology
- To know the applications of fuzzy sets, fuzzy relations, fuzzy algebra and fuzzy topology in science and technology.

Course Content:

1. Fuzzy sets and L-fuzzy sets.
2. Operations on fuzzy and L-fuzzy sets. Fuzzy complements.
3. Fuzzy Intersections: t-norms.
4. Fuzzy Unions: t-conorms and their applications of operations.
5. Fuzzy relations: Crisp versus Fuzzy Relations, Binary Fuzzy Relations and Binary Relations on a single set, Fuzzy Equivalence, Compatibility and Ordering Relations. Fuzzy morphisms. Sup-i and Inf-w_i of Fuzzy Relations.
6. Fuzzy Relation Equations: General discussion, Partitioning Problem and solution Method, Fuzzy Relation Equations based on Sup-i and Inf-w_i compositions.
7. Fuzzy Algebraic Structure: Fuzzy Semi-groups, Groups, Rings and Modules, Fuzzy Lattice.
8. Fuzzy Topological Structure: Fuzzy Open and Closed sets, Fuzzy neighborhood and Fuzzy Closure, Fuzzy Compactness, Fuzzy Separation Axioms.
9. Applications of Fuzzy Sets: Approximate Reasoning, Fuzzy Relational Inference, Fuzzy Controllers, Efficiency and Effectiveness of inference schemes, Functional Approximation capabilities, Fuzzy Decision Making.

Suggested Readings:

1. J. Klir and Boe Yen (1995), *Fuzzy Sets and Fuzzy Logic; Theory and Applications*, Prentice Hall P T R Upper Saddle River, New Jersey 07458.
2. J. N. Mordeson, K. R. Bhutani and A. Rosenfeld (2005), *Fuzzy Group Theory*, Springer.
3. Liu Ying-Ming and Luo Mao-Kang (1997), *Fuzzy Topology*, world Scientific Publishing Co. Pte. Ltd, Singapore 912805.

AMath 514: Lattice Theory and Applications

Learning Objectives:

- To be familiar with lattice theory.
- To acquaint with sub lattice, lattice homomorphism and ideal of lattice.
- To study further aspects of quotient lattice.
- To know the basic definitions lattice theory.
- To know the application of lattice theory and Boolean Algebra.

Course Content:

1. Relation, equivalence relation, order relation, ordered sets (posets), order preserving maps, isotone, down sets, up sets, ordered sets of down sets.
2. Lattices as an ordered set and as an algebra and the relation between them, complete lattice, semi lattice.
3. Sub lattice, convex sub lattice, ideals, prime ideals, filters, maximal ideals.
4. Modular and distributive lattices, characterization of modular and distributive lattices.
5. Homomorphism of lattices, epimorphism, isomorphism.
6. Congruence and quotient lattices.
7. Complements and pseudo complementation.
8. Boolean Algebra: Boolean lattice and Boolean algebra, stone’s representation theorem for Boolean algebra, Boolean functions and Circuit network, etc.

Suggested Readings:

1. G. Birkhoff (1967), *Lattice Theory,* Amer. Math. Soc. Coll. Publications
2. G. Gratzer (2001), *General Lattice Theory,* Birkhäuser
3. D. K. Rutherford (1965), *Introduction to Lattice Theory,* Hafner Pub. Co
4. V. K. Khanna (2017), *Lattice and Boolean Algebras,* Vikas Publishing House

AMath 515: Mathematical Statistics

Learning Objectives:

- To demonstrate knowledge and properties statistical models in common use.
- To understand the basic principles underlying statistical inference (estimation and hypothesis testing).
- To be able to construct tests and estimators, and derive their properties.
- To demonstrate knowledge of applicable large sample theory of estimators and tests.

Course Content:

1. ***Introduction to Mathematical Statistics:*** Data and probability models, parameters and statistics, Bayesian models, statistical inference as a decision problem, prediction, sufficient statistics, exponential families of probability models.
2. ***Estimation Methods:*** Least squares, weighted lease squares, method-of-moments (MOM), maximum likelihood, Bayes, m-estimation, estimation algorithms
3. ***Performance Measurement and Optimization:*** Bayes procedures, minimax procedures, constrained optimization, robustness criteria.
4. ***Hypothesis Testing and Confidence Regions:*** Neyman-Pearson lemma, UMP tests, monotone likelihood ratio models, confidence bounds, confidence intervals/regions.
5. ***Asymptotics:*** Consistency, asymptotic normality, MLEs in exponential families, m-estimators, efficiency, limiting posterior distribution
6. ***Multiparameter Statistical Inference:*** Gaussian linear models, large sample tests, generalized linear models.

Suggested Readings:

1. Peter J. Bickel and Kjell A. Doksum (2015), *Mathematical Statistics: Basic Ideas and Selected Topics*, 2nd edition, Chapman and Hall/CRC.
2. K. M. Ramachandran and Chris P. Tsokos (2009), *Mathematical Statistics with Applications*, Elsevier.
3. Dennis Wackerly, William Mendenhall, and Richard L. Scheaffer (2008), *Mathematical Statistics with Applications*, 7th edition, Thomson Brooks/Cole.

AMath 516: Machine Learning

Learning Objectives:

- To grasp the foundations of machine learning, encompassing supervised, unsupervised, and reinforcement learning.
- To delve into the intricate world of performance analysis, mastering concepts like Linear Regression, Logistic Regression, Naive Bayes, Loss Functions, Accuracy, and ROC curve.
- To Implement diverse models through Scikit-learn, they will gain proficiency in regression techniques, spanning Linear Regression, Decision Trees, and Logistic Regression, while tackling overfitting challenges.
- To understand the intricacies of Support Vector Machines, k-Nearest-Neighbor algorithms, Bayesian learning, and unsupervised learning techniques like k-means, adaptive hierarchical clustering, and Gaussian mixture models

Course Content:

1. ***Introduction****:* Machine Learning (ML), Supervised, Unsupervised, Reinforcement learning.
2. **Machine Learning and performance analysis:** Linear Regression, Logistic Regression, Naive Bayes, Loss Functions, Accuracy, ROC curve, implementation of different Models (Scikit-learn).
3. ***Regression:*** Linear regression, Decision trees, overfitting, Logistic Regression.
4. ***Neural network****:* Perceptron, multilayer network, backpropagation, introduction to deep neural network
5. ***Support Vector Machines & k-Nearest-neighbor algorithm:*** Maximum margin linear separators. Quadratic programming solution to finding maximum margin separators. Kernels for learning non-linear functions, k-Nearest-neighbor algorithm.
6. ***Bayesian Learning****:* Probability theory and Bayes rule. Naive Bayes learning algorithm. Parameter smoothing. Generative vs. discriminative training.
7. ***Clustering and Unsupervised Learning****:* Clustering: k-means, adaptive hierarchical clustering, Gaussian mixture model

Suggested Readings:

1. Andrew NG (2018), *Machine Learning Yearning,* deeplearning.ai
2. Sebastian Raschka & Vahid Mirjalili (2015), *Python Machine Learning*, Packt
3. Peter Flach (2012), *Machine Learning: The Art and Science of Algorithms that Make Sense of Data*, Cambridge University Press

AMath 517: Data Analytics

Learning Objectives:

- To understand data analytics fundamentals for a comprehensive foundation in data analysis concepts and tools.
- To explore data understanding and preprocessing, understanding structured and unstructured data, data analysis processes, handling missing data, data processing and visualization, encompassing Pandas for data manipulation and visualization techniques like Seaborn and map plotting.
- To develop skills for Mathematical and scientific applications using Numpy and Scipy focusing on N-dimensional arrays and data processing.
- To delve into web data analysis, learning web scraping, data combination, transformation, and string manipulation.

Course Content:

1. ***Fundamentals for Data Analysis:*** Python data structures, Control statements, Functions, Object Oriented programming concepts using classes, objects and methods, Exception handling, Implementation of user-defined Modules and Package, File handling in python.

2. ***Introduction to Data Understanding and Preprocessing:*** Knowledge domains of Data Analysis, understanding structured and unstructured data, Data Analysis process, Dataset generation, Importing Dataset: Importing and Exporting Data, Basic Insights from Datasets, Cleaning and Preparing the Data: Identify and Handle Missing Values.

3. ***Data Processing and Visualization:*** Data Formatting, Exploratory Data Analysis, Filtering and hierarchical indexing using Pandas. Data Visualization: Basic Visualization Tools, Specialized Visualization Tools, Seaborn Creating and Plotting Maps.

4. ***Mathematical and Scientific applications for Data Analysis:*** Numpy and Scipy Package, Understanding and creating N-dimensional arrays, Basic indexing and slicing, Boolean indexing, Fancy indexing, Universal functions, Data processing using arrays, File input and output with arrays.

5. ***Analyzing Web Data:*** Data wrangling, Web scrapping, Combing and merging data sets, Reshaping and pivoting, Data transformation, String Manipulation, case study for web scrapping.

6. ***Model Development and Evaluation:*** Introduction to machine learning- Supervised and Unsupervised Learning, Model development using Linear Regression, Model Visualization, Prediction and Decision Making, Model Evaluation: Over-fitting, Under-fitting and Model Selection.

Suggested Readings:

1. David Ascher and Mark Lutz (2004), *Learning Python*, O’Reilly Media.
2. Reema Thareja (2017), *Python Programming using Problem Solving approach*, Oxford University press
3. Wes Mckinney (2017), *Python for Data Analysis*, O’Reilly Media.

AMath 518: Integral Equations with Applications

Learning Objectives:

- To provide instruction in techniques used in solving integral equations commonly encouraged in natural sciences.
- To identify integral equations by their types.
- To convert applied research models by ODEs into models by integral equations and vice versa.
- To solve integral equations of different types and basic theory concerning the existence and uniqueness of the solutions.

Course Content:

1. Classification of linear integral equations, conversion of initial value problem into Volterra integral equation, conversion of boundary value problem into Fredholm integral equation, separable kernels, iterated kernels, eigenvalues and eigenfunctions, solution of homogeneous and general Fredholm integral equations of second kind with separable kernels.
2. Solution of Fredholm and Volterra integral equations by successive approximation method, successive substitution method, Adomian decomposition method, modified decomposition method, Neumann series. Integrodifferential equations, solution of Fredholm and Volterra integrodifferential equations by Adomian decomposition method.
3. Resolvent kernel, solution of Fredholm and Volterra integral equations with the help of resolvent kernel, Fredholm alternative, Fredholm fundamental theorems, solution of Fredholm integral equation of second kind by using Fredholm first theorem.
4. Integral equations with symmetric kernels: orthogonal and orthonormal system of functions, fundamental properties of eigenvalues and eigenfunctions for symmetric kernels, Hilbert-Schmidt theorem, solution of Fredholm integral equations of second kind by using Hilbert-Schmidt theorem.
5. Convolution type kernels, application of Laplace transform to solutions of Volterra integral equations, application of Fourier transform to solutions of singular integral equations, Green's function: definition, construction of Green's function and its use in solving boundary value problems.

Suggested Readings:

1. Abdul Majid Wazwaz (2011), *Linear and Nonlinear Integral Equations: Methods and Applications*, Springer,
2. M.D. Raisinghania (2010), *Integral Equations and Boundary Value Problems*, S. Chand & Co. Ltd.
3. Shanti Swarup (2010), *Integral Equations*, Krishna Prakashan Media (P) Ltd.

Preliminary Courses

PAMath 519: Theory of Number Systems

Learning Objectives:

- To find the greatest common divisor and least common multiple of a pair of natural numbers, and finding the linear form of the greatest common divisor.
- To learn prime factorization.
- To understand techniques of solving linear congruence's and systems of simultaneous linear congruence's.
- To determine whether a quadratic congruence has solutions, and if so, finding them.

Course Content:

1. Real numbers, classifications of real numbers, theorems on rational and irrational numbers, Archimedean property of real numbers.
2. Sets: Bounds of sets, supremum, infimum of sets, Nest of intervals, neighborhood , limit point, interior point, Derived set, open sets, closed sets, De Morgan's law of sets, theorems on open and closed sets.
3: Sequences: Definition, Subsequences, Bounded and unbounded sequences, limit of sequence, Convergence, Cauchy sequence.
4. Series: Convergence of series, Test of convergence of series.
5. Divisibility, Division algorithm, Prime number, Fundamental theorem of Arithmetic and its consequences.
6. Congruences, quadratic congruence. Congruences of higher degrees, Complete residue system.
7. Linear Diophautine equations. Fermats theorem. Wilson's theorem.
8. Arithmetic functions $\varphi(n), \mu(n), d(n), \sigma_{\alpha}(n)$.

Suggested Readings:

1. William R. Parzynski, Philip W. Zipse, Introduction to Mathematical Analysis, McGraw-Hill, 1982
2. Robert G. Bartle, Donald R. Sherbert, Introduction to Real Analysis 4^{th} edition, John Wiley & Sons. Inc.
3. David M. Burton : Elementary Number theory, Tata McGraw-Hill, 6^{th} Edition, 2007.
4. Underwood Dudley : Elementary Number Theory, W. H. Freeman and Company, New York. Second Edition.

PAMath 520: Calculus with Analytical Geometry

Learning objectives:

- To develop techniques of derivatives and apply them in determining maxima, minima, tangents, normals, asymptotes, curvature, envelops and evolutes.
- To acquire techniques of integrals and apply these in finding the length of a plane curve, area under a curve, area between two curves, volumes, area of a surface of revolution.
- To develop a working knowledge of the gradient, including its relationship to level curves (or surfaces), directional derivatives.
- To gain the ability to set up and solve optimization problems involving several variables, with or without constraints.
- To understand of line integrals for work and flux, surface integrals for flux, general surface integrals and volume integrals including Green's and Stokes' theorems.
- To create interest about the basic need to study calculus by demonstrating and explaining some real life events or features.

Course Content:

A. Calculus of Single Variable:

1. Functions and Graphs including polar coordinates, limits and continuity.
2. Derivative and techniques of differentiation, successive differentiation, Application of derivatives, Increasing and decreasing, extrema of functions, tangent and normal.
3. Integrals, techniques of integration, Applications of definite integrals: Area under a curve, area between two curves; volumes etc.

B. Calculus of Multi Variable:

1. Functions, limits, continuity of function of multivariable.
2. Partial derivatives, directional derivatives, gradient tangent planes, normal vector, applications of partial derivatives: Maxima and Minima.
3. Multiple integrals, Jacobians, Applications of multiple integrals: Area, volume, surface area, etc.
4. Vector calculus: Vector differentiation, gradient, divergence and curl, vector integration, Line integrals, Surface integrals, Volume integrals, Green's theorem, divergence theorem, Stokes's theorem.

Suggested Readings:

1. James Stewart (2005), *Calculus*, Thomson learning INC.
2. Anton, Bivens, Davis (2016), *Calculus Early Transcendentals*, Wiley.
3. Robert T. Smith, Ronald B. Minton (2003), *Multivariable Calculus*, McGraw-Hill.
4. Murray R. Spiegel, Seymour Lipschutz, Dennis Spellman (2009), *Vector Analysis*, McGraw-Hill.

PAMath 521: Linear Algebra

Learning objectives:

- To impart concept and techniques of basic linear algebra
- To develop computational proficiency
- To understand the theories and proofs

Course Content:

1. ***Matrices:*** Introduction, types of matrices, matrix algebra, inverse of a matrix, similar matrices, symmetric, orthogonal and Hermitian matrices.
2. ***System of linear equations:*** Consistency and inconsistency of system, graphical solutions, elementary matrices, Gauss elimination method, Gauss-Jordan method, Cramer's rule, applications.
3. ***Vector spaces:*** Introduction, definition and examples of vector spaces, subspaces, linear independence, basis, dimension and direct sums.
4. ***Linear transformations on vector spaces:*** Linear transformation, the matrix of a linear transformation, the kernel and image of a linear transformation, rank and nullity of a linear transformation, k-isomorphism and non-singular linear transformations, applications to linear equations and the rank of matrices.
5. ***Inner product spaces:*** Introduction to three dimensional geometry, Euclidean and unitary spaces, orthogonality and Gram-Schmidt process.
6. ***Diagonalization of matrices:*** Introduction, eigenvalues and eigenvectors, diagonalization of matrices, the minimum polynomial of a matrix, the Cayley-Hamilton theorem, the diagonalization of symmetric matrices, Jordan canonical form.

Suggested Readings:

1. H. Anton (2019), *Elementary Linear Algebra*, Wiley.
2. S. Lang (2004), *Linear Algebra*, Springer.
4. R. Larsson (2012), *Elementary Linear Algebra*, Cengage Learning.

PAMath 522: Ordinary Differential Equations

Learning Objectives:

- To acquaint with differential equations and simple solutions
- To form simple Mathematical models and analyze the behavior of the solutions
- To explain first-order and higher-order differential equations, along with the methods of solutions and their applications
- To learn series solutions of linear equations as well as their properties

Course Content:

1. ***Introduction:*** Formation of ODEs and classification of ODEs. Modeling of physical problems by ODEs.
2. ***First order ODEs*****:** Classification of first order ODEs, Separable, Homogeneous, Linear, Exact equations, Integrating factors, Bernoulli equation, Riccati equation.
3. ***First order higher degree equations:*** Solvable for x, y and p, Clairaut's equation, singular solutions, orthogonal and oblique trajectories.
4. ***Higher order linear ODEs:*** Higher order linear homogeneous equation with constant coefficients, Reduction of orders, basic theorems.
5. ***Linear Nonhomogeneous ODEs:*** Linear nonhomogeneous ODEs with constant coefficients, Method of undetermined coefficients, Method of variation of parameters, operator method.
6. ***System of linear ODEs:*** Method of elimination, Euler's method, Matrix method, Homogeneous and Non-homogeneous linear system.
7. ***Series Solutions of ODEs:*** Power series, Taylor series, Frobenius method.

Suggested Readings:

1. Dennis Zill (2019), *A First Course in Differential Equations with Modeling Applications*, Cengage Learning India Pvt. Ltd.
2. Rainville & Bedient (1996), Elementary Differential Equations, Prentice Hall.
3. Clay C. Ross (2010), *Differential Equations*, Springer.

PAMath 523: Partial Differential Equations

Learning Objectives:

- To introduce PDEs and its applications.
- To explain fundamental concepts of PDE theory.
- To discuss analytical methods for solving PDEs.

Course Content:

1. ***First order Equations:*** Introduction, Classification- Linear, quasi-linear, nonlinear, homogeneous and non-homogeneous PDE, Origin of PDE, Formation of PDE.
2. ***Hyperbolic Differential Equations:*** Occurrence of the wave equation, IVP- D'Alembert solution IBVPs. method of separation of variables, method of Eigen function, method of characteristics, method of Green's functions, Fourier transformation method, Laplace transformation method.
3. ***Parabolic Differential Equation:*** Occurrence of the heat/diffusion equation, boundary conditions. IBVPs, method of separation of variables, method of Eigen function, method of green's functions, Fourier transformation method, Laplace transformation method.
4. ***Elliptic Differential Equation:*** Occurrence of Laplace equation and Poisson equation, boundary conditions, BVPs, method of separation of variables, method of Eigen function, method of green's functions.
5. ***Boundary Value Problems:*** Solution of 1D wave equation in cylindrical and spherical polar co-ordinates, solution of Laplace equation in cylindrical and spherical polar co-ordinates.

Suggested Readings:

1. K. Sankara Rao (2011), *Introduction to Partial Differential Equations*, PHI Learning Pvt. Ltd.
2. Christian Constanda (2010), *Solution Techniques for Elementary PDEs*, Chapman & Hall.
3. Richard Haberman (2012), *Elementary Applied Partial Differential Equations: With Fourier Series and Boundary Value Problems*, Pearson
4. M. D. Raisinghania (2020), *Ordinary and Partial Differential Equations*, S. Chand & Co. Pvt. Ltd.
5. Paul DuChateau, D. Zachmann (1986), *Partial Differential Equations*, Schaum's Outline Series.

PAMath 524: Special Functions and Integral Transforms

Learning Objectives:

- To develop a basic understanding of a range of mathematical tools with emphasis on engineering applications
- To develop skills, think quantitatively and analyze problems critically
- To analyze properties of special functions by their integral representations and symmetries
- To determine properties of different integral transforms which may be applied to solve various physical problems

Course Content:

Part A: Special Functions

1. ***Bessel Functions:*** Solution of Bessel's Equation, Generating function Recurrence relation, values of Bessel function, orthogonality, Modified Bessel function.
2. ***Legendre Polynomials:*** Solution of Legendre equation, Generating function, Recurrence relation, Rodrigue's formula and orthogonality of Legendre polynomials.
3. ***Hermite Polynomials:*** Solution of Hermite equation, Integral and recurrence formula, orthogonality.

Part B: Integral Transforms

4. ***Laplace Transforms:*** Laplace transforms and its existence conditions, Basic properties of Laplace transforms, Convolution theorem, Differentiation and Integration of Laplace transforms,
5. ***Inverse Laplace Transforms:*** Inverse Laplace transforms, Application of inverse Laplace transforms to ODEs.
6. ***Fourier Series and Fourier Transforms:*** Fourier series; Fourier transforms, Basic properties
7. ***Applications of Fourier Transforms:*** Application of Fourier transforms to ODEs, Fourier sin and cosine transform.

Suggested Readings:

1. Jeffreys (2000), *Methods of Mathematical Physics*, Cambridge University Press.
2. L. Debnath & D. Bhatta (2014), *Integral Transforms and Their Applications*, Chapman and Hall/CRC.
3. J. K. Goyal, K. P. Gupta (2015), *Integral Transforms*, Pragati Prakashan.

AMath 530: Viva voce

Full Marks: 50

- Viva voce on courses taught in the MSAM Program.

AMath 531: Report

Full Marks: 50 (*Report Writing: 35 + Report Presentation: 15*)

- A particular topic on Mathematics to be given by the concerned supervisor. A student has to present his/her report in front of the MSAM program coordination committee.